Disconnected contribution to the muon $g-2$ HVP

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University of Utah

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Fermilab-HPQCD-MILC

Fermilab Lattice and MILC:

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- Pietro Butti
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- Carleton DeTar
- Aida El-Khadra
- Elvira Gámiz
- Steven Gottlieb
- Anthony Grebe
- Leon Hostetler
- William Jay
- Hwancheol Jeong
- Andreas Kronfeld
- Shaun Lahert
- Michael Lynch
- Andrew Lytle
- Ethan Neil
- Curtis Peterson
- James Simone
- Jacob Sitison
- Ruth Van de Water
- Alejandro Vaquero
- Shuhei Yamamoto

HPQCD:

- Christine Davies
- Peter Lepage
- Craig McNeile
- Gaurav Ray

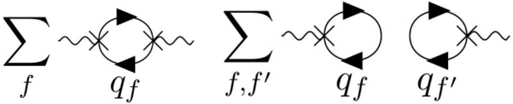
Lattice HVP

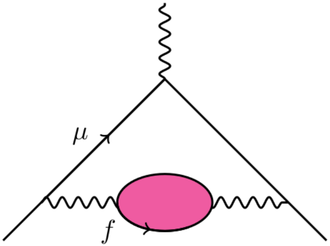
In time-momentum representation[1]:

$$a_\mu^{\mathrm{HVP,LO}} = 4\alpha^2 \int_0^\infty \mathrm{d}t\, C(t)\tilde{K}(t),$$

$$C(t) = \frac{1}{3}\sum_{\mathbf{x},k}\left\langle J^k(\mathbf{x},t)J^k(0)\right\rangle$$

$$J^\mu(x) = \sum_f Q_f \bar{q}_f(x)\gamma^\mu q_f(x)$$



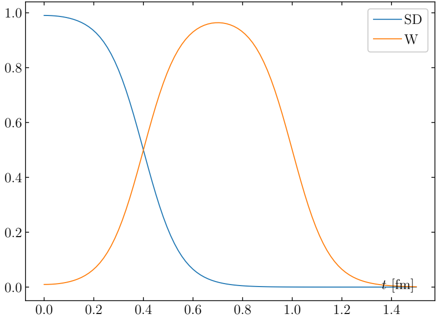


[1]D. Bernecker and H. B. Meyer, Eur. Phys. J. A, 47.11, 148 (2011).

Window observables[2]

$$a_\mu^{\mathrm{win}(t_0,t_1,\Delta)} = 4\alpha^2 \int_0^\infty \mathrm{d}t\, C(t)\tilde{K}(t)\mathcal{W}\left(t,t_0,t_1,\Delta\right)$$

$$\mathcal{W}\left(t,t_0,t_1,\Delta\right) = \frac{1}{2}\left[\tanh\left(\frac{t-t_0}{\Delta}\right) - \tanh\left(\frac{t-t_1}{\Delta}\right)\right] + (t \to -t)$$



[2] T. Blum et al., Phys. Rev. Lett. 121.2, 022003 (2018).

The disconnected contribution

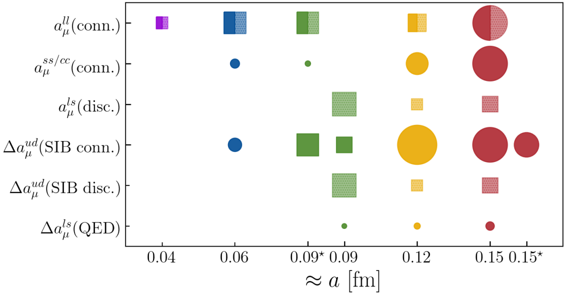
$$a_\mu^{\mathrm{HVP,LO}} = a_\mu^{ll}(\mathrm{conn.}) + a_\mu^{ss}(\mathrm{conn.}) + a_\mu^{cc}(\mathrm{conn.}) + a_\mu^{bb}(\mathrm{conn.}) + \dots$$
$$+ a_\mu(\mathrm{disc.}) + \Delta a_\mu(\mathrm{SIB}) + \Delta a_\mu(\mathrm{QED})$$

Previously covered:

- Connected SD & W (Shaun Lahert, Wednesday 12:15)
- Light-quark connected LD & full (Michael Lynch, Wednesday 12:35)
- QED corrections (Craig McNeile, poster)

Next talk, Jake Sitison discusses SIB corrections.

Set up



- $N_f = 2+1+1$ HISQ sea quarks
- Common bootstrap scheme
- Renormalize with $Z_V^{\mathrm{RI-SMOM}}$
- Bayesian model averaging with variations
  - FV: NLO/NNLO χPT, chiral model (CM)
  - with and without taste breaking (TB)
- Only 3 ensembles for this subanalysis
  - $N_{\mathrm{conf}} = 700 - 1700$
- All shown results are blinded

Bayesian model averaging[3,4] (BMA)

Given set of data analysis choices M and raw correlators D:

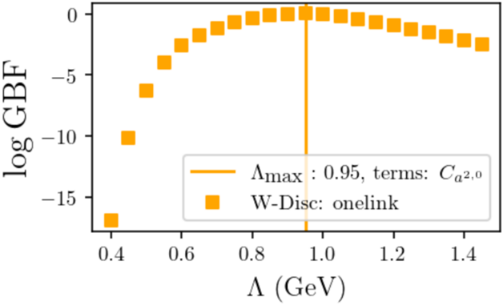
$$\mathrm{pr}\left(M|D\right) \equiv \mathrm{pr}\left(M\right) \exp\left[-\frac{1}{2}\left(\chi^2_{\mathrm{data}} + 2N_{\mathrm{param}} + 2N_{\mathrm{cut}}\right)\right]$$

For this subanalysis, $N_{\mathrm{cut}} = 0$. Mean and variance are

$$\begin{aligned}
\langle a_\mu \rangle &= \sum_{n=1}^{N_{\mathrm{model}}} \langle a_\mu \rangle_n \, \mathrm{pr}\left(M_n|D\right), \\
\sigma^2_{a_\mu} &= \sum_{n=1}^{N_{\mathrm{model}}} \sigma^2_{a_\mu,n} \, \mathrm{pr}\left(M_n|D\right) + \underbrace{\sum_{n=1}^{N_{\mathrm{model}}} \langle a_\mu \rangle_n^2 \, \mathrm{pr}\left(M_n|D\right) - \langle a_\mu \rangle^2}_{\text{“systematic”}}
\end{aligned}$$

[3] E. T. Neil and J. W. Sitison, Phys. Rev. E, 108.4, 045308 (2023).
[4] E. T. Neil and J. W. Sitison, Phys. Rev. D, 109.1, 014510 (2024).

Choosing a Λ (empirical Bayes)



$$a_\mu(a) = a_\mu \left(1 + \sum_{i \text{ even}} \sum_{j=0}^{4} c_{ij}\alpha_s^j (a\Lambda)^i\right)$$

c_{20}, c_{40} relevant (one-link); $\Lambda = 0.9$ GeV

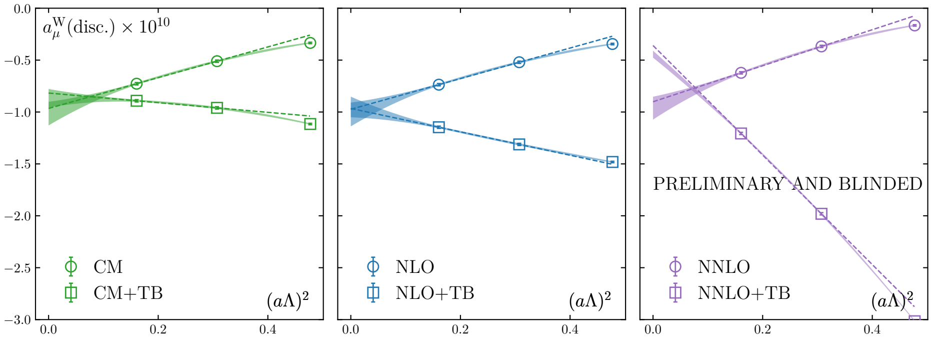
Continuum-limit extrapolations

Empirical Bayes suggests fits for W:

$$a_\mu(a) = a_\mu^{\rm cont}\left(1+\sum_{i=1}^{n} c_i (a\Lambda)^{2i}\right).$$

- Always at least $a_\mu^{\rm cont}$ and c_1 (no prior)
- Diffuse priors 0(2) otherwise
- Try NLO, NNLO, CM correction schemes
- Altogether $3\times2\times2=12$ models

Example W extrapolation



Bands show fit going into BMA. Dotted lines show $\mathcal{O}\left(a^2\right)$ fit to finest two points.

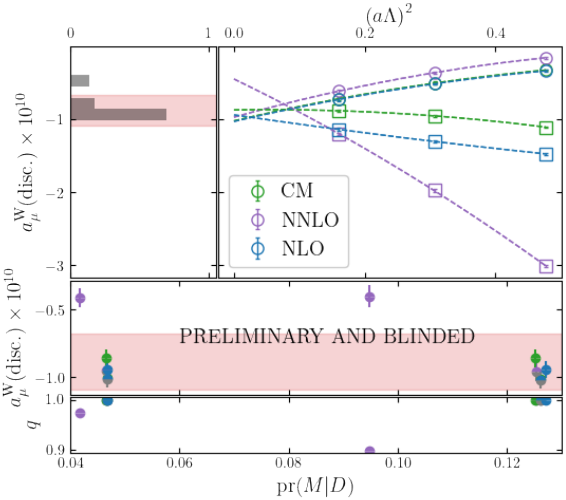
Finer spacing will help elucidate NNLO+TB

Bootstrap strategy

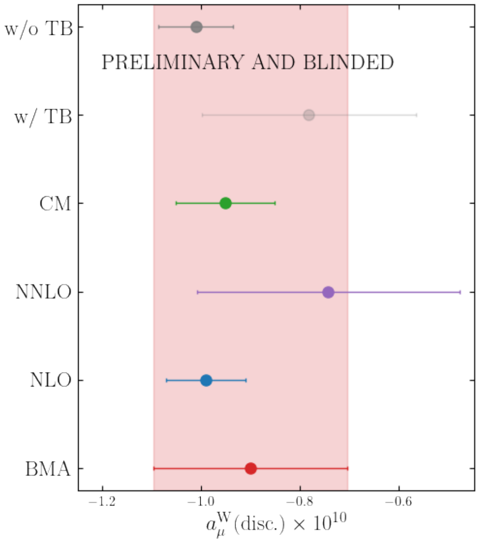
In each of $N_{\text{boot}} = 500$ samples:

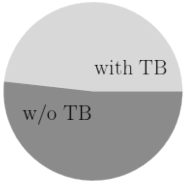
1. Draw Gaussian w_0, w_0/a, Z_V (common sample to all subanalyses)
2. Integrate $C(t)K(t)$ for specified window
3. Blind and apply systematic corrections
4. Try every fit model M
5. For BMA, use $\mathrm{pr}\,(M|D)$ computed from naive (no resampling) data set
6. Report median; middle 68% is uncertainty

BMA for W, squares have TB)



BMA for W





$$\mathrm{pr}\,(S|D) = \sum_{M\in S} \mathrm{pr}\,(M|D)$$

$\sigma(\mathrm{stat.}) = 7.7\%$

$\sigma(\mathrm{syst.}) = 22\%$

SD analysis

- Distance at short enough scales for FV to be negligible
- Anyway the EFTs don't apply at these scales
- Only 3 data with high curvature
- Hence they do not tolerate diffuse priors

We try (no BMA)

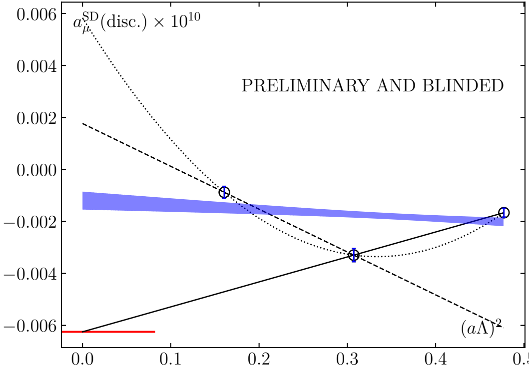
- Simple linear (in a^2) solves to coarsest and finest two
- Simple quadratic solve to all three
- Linear fit to all three

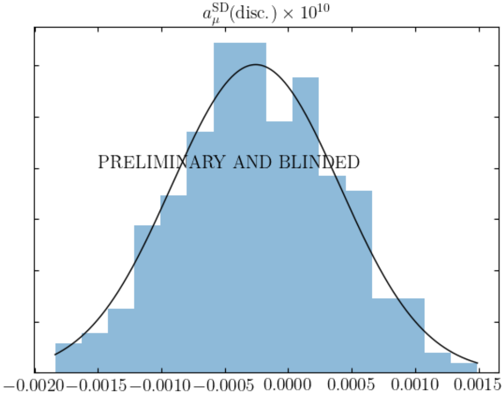
Also show a pQCD comparison using `rhad`[5]

[5] R. V. Harlander and M. Steinhauser, Computer Physics Communications, 153.2, 244–274 (2003).

SD analysis

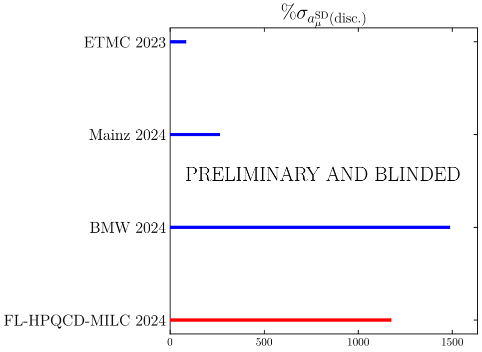
Use difference at $a = 0$ to estimate systematic error, bounds $a_\mu^{\mathrm{SD}}(\mathrm{disc.})$

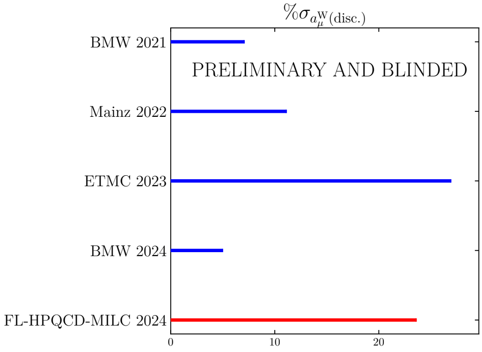




$\sigma(\mathrm{syst.}) \sim 1200\%$ $\sigma(\mathrm{stat.}) \sim 130\%$

Precision compared to recent literature





Uncertainty in both cases dominated by systematics

Summary and outlook

- Disconnected SD and W will be unblinded soon
- Both observables will profit from finer spacings
- SD compatible with 0 as with BMW and ETMC
- LD and full in progress

Thanks for listening