Topological Charge and Cooling Scales in Pure SU(2) Lattice Gauge Theory

David A. Clarke

Florida State University

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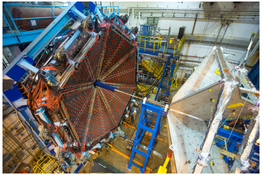
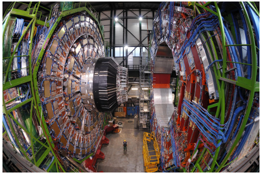
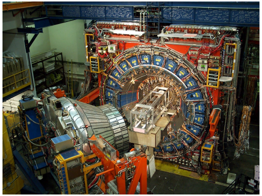
B. A. Berg and D. A. Clarke, Phys. Rev. D **97** 054506 (2018)

Background

- Lattice: investigate physical observables non-perturbatively
- Investigation of SM, NP, hadron properties, confinement, etc. require precise calculations of physical observables
- In continuum limit, one predicts dimensionless mass ratios m/m_0
- Choosing the reference m_0 is called scale setting
- Scale setting is a source of systematic uncertainty $\Rightarrow$ it is useful to investigate scales that can achieve small statistical error bars







Context of project

- Lüscher introduces the gradient flow[1], suggests gradient scale as a new reference scale; scale setting gains renewed interest
- Bonati and D'Elia[2] suggest standard cooling[3] can be used similarly for scale setting; advantage in computational efficiency
- We verify[4] this works in pure SU(2)
- This project:
  1. Investigate whether there is any dependence of the cooling scales on topological charge
  2. Obtain a new estimate for topological susceptibility

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[1] M. Lüscher, J. High Energy Phys. 2010 (2010).

[2] C. Bonati and M. D'Elia, Phys. Rev. D, 89 (2014).

[3] B. A. Berg, Phys. Lett. B, 104, 475 (1981).

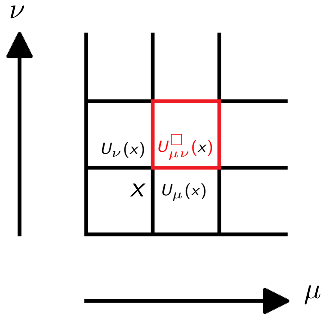
[4] B. A. Berg and D. A. Clarke, Phys. Rev. D, 95 (2017).

Pocket dictionary for pure SU(2) LGT

- 4D space-time with Euclidean metric and periodic BCs
- Hypercube of volume $(aN)^4$
- Sites $x = (an_1, an_2, an_3, an_4)$ with n_4 in time direction
- Regularization through lattice spacing a
- $\beta = 4/g^2 \to \infty$ controls continuum limit $a/m_0 \to 0$
- Link variables $U_\mu(x) = e^{-aA_\mu(x)} \in \mathrm{SU}(2)$ on links
- Plaquette $U^\square_{\mu\nu}(x)$
- Wilson action:

$$S = \beta \sum_{x,\mu<\nu} \left(1 - \tfrac{1}{2} \operatorname{tr} U^\square_{\mu\nu}(x)\right)$$

$$\approx -\frac{\beta}{8} \sum_x a^4 \operatorname{tr} F_{\mu\nu}(x) F_{\mu\nu}(x)$$



Topological charge on the lattice

Discretize

$$Q = \frac{1}{32\pi^2} \int d^4x \, \epsilon_{\mu\nu\rho\sigma} \, \mathrm{tr} \, F_{\mu\nu} F_{\rho\sigma}$$

as

$$Q = \frac{1}{2^9 \pi^2} a^4 \sum_x \sum_{\mu\nu\rho\sigma=\pm 1}^{\pm 4} \epsilon_{\mu\nu\rho\sigma} \, \mathrm{tr} \, U^{\square}_{\mu\nu}(x) U^{\square}_{\rho\sigma}(x).$$

Topological susceptibility:

$$\chi = \frac{1}{V} \langle Q^2 \rangle$$

- Provides information about distribution of Q
- Phenomenological interest (η - η' mass difference)
- Must smooth out local fluctuations before measuring

Cooling

We "smooth" using standard cooling (locally minimize action)

$$V_\mu(x,n_c)=\frac{V_\mu^{\sqcup}(x,n_c-1)}{\sqrt{\det V_\mu^{\sqcup}(x,n_c-1)}},\qquad V_\mu(x,0)=U_\mu(x).$$

- Suppresses local fluctuations
- Extract Q and χ from cooled configurations, after these quantities become "metastable" (for large enough β)

Given target y and operator E, a cooling scale $L=\sqrt{n_c}$ is defined by

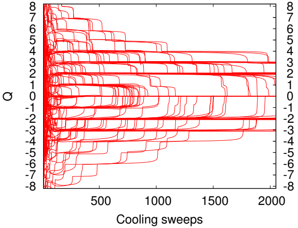
$$y=n_c^2\left\langle E_{n_c}\right\rangle.$$

As reference, take the cooling scale L_{10}[5].

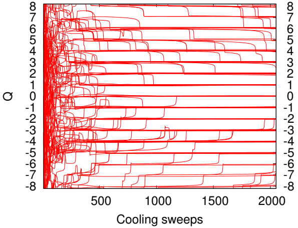
[5] B. A. Berg and D. A. Clarke, Phys. Rev. D, 95 (2017).

Metastability of cooling trajectories

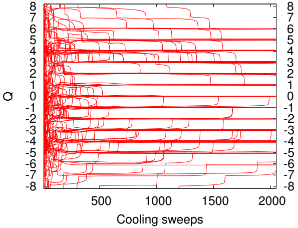
$\beta = 2.300,\ N^4 = 16^4$



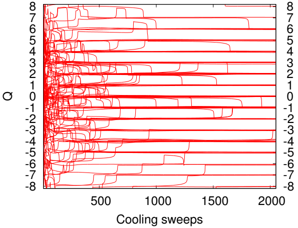
$\beta = 2.430,\ N^4 = 28^4$



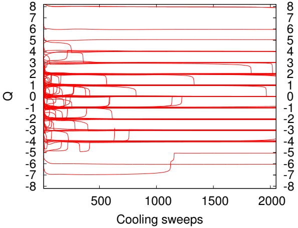
$\beta = 2.510,\ N^4 = 28^4$



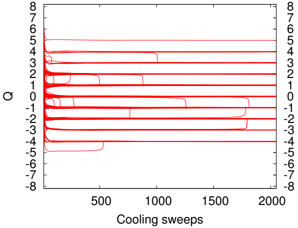
$\beta = 2.620,\ N^4 = 40^4$



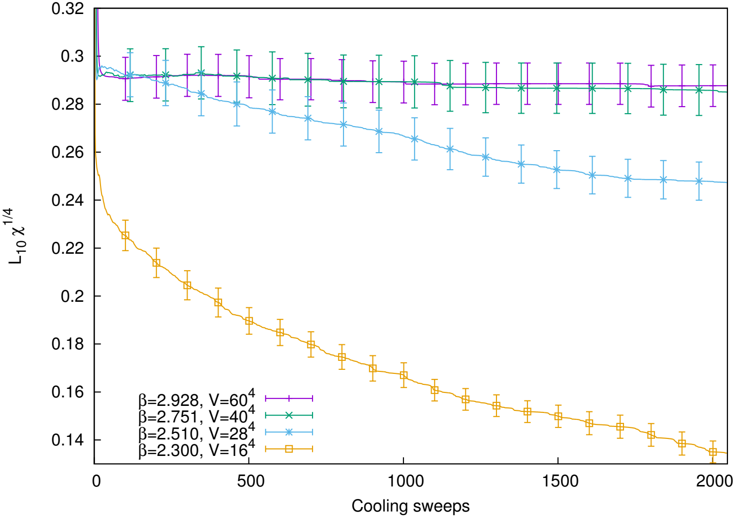
$\beta = 2.751,\ N^4 = 40^4$



$\beta = 2.928,\ N^4 = 60^4$



Stabilization of χ



χ and Q metastable for $\beta \gtrsim 2.751$

Prediction for topological susceptibility

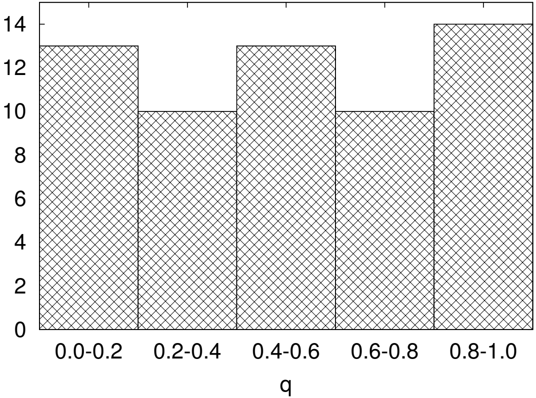
$$\chi^{1/4}/\sqrt{\sigma} = 0.4446(82) \quad \text{for} \quad n_c = 1000$$
$$\chi^{1/4}/\sqrt{\sigma} = 0.4655(87) \quad \text{for} \quad n_c = 100$$

$\chi^{1/4}/\sqrt{\sigma}$	q_{1000}	q_{100}
0.501(45)[6]	0.31	0.44
0.528(21)[7]	0.00	0.01
0.480(23)[8]	0.32	0.56
0.4831(56)[9]	0.01	0.09
0.4745(63)[9]	0.07	0.40
0.4742(56)[9]	0.06	0.40

[6] P. De Forcrand, M. G. Perez, and I.-O. Stamatescu, Nucl. Phys. B, 499, 409 (1997).
[7] T. DeGrand, A. Hasenfratz, and T. G. Kovacs, Nuclear Physics B, 505, 417 (1997).
[8] B. Allés, M. D'Elia, and A. Di Giacomo, Phys. Lett. B, 412, 119 (1997).
[9] B. Lucini and M. Teper, J. of High Energy Phys. 2001, 050 (2001).

Dependence of cooling scale on Q

- Compare $L(Q_1)$ with $L(Q_2)$ using Student difference test
- Found $L(Q)$ statistically compatible with $L(-Q)$
- Found scales with $|Q| > 1$ to be statistically compatible
- Therefore combine into bins $Q = 0$, $|Q| = 1$, and $|Q| \geq 2$



Conclusions

- For large enough β and N, standard cooling can be used to obtain metastable topological sectors
- Best estimate for topological susceptibility:

$$\chi^{1/4}/\sqrt{\sigma} = 0.4446(82),$$

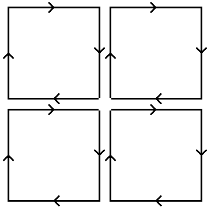
  which is surprisingly close to past results
- Within our statistics, we find no evidence of correlations between cooling scales due to topological charges

Thanks for listening

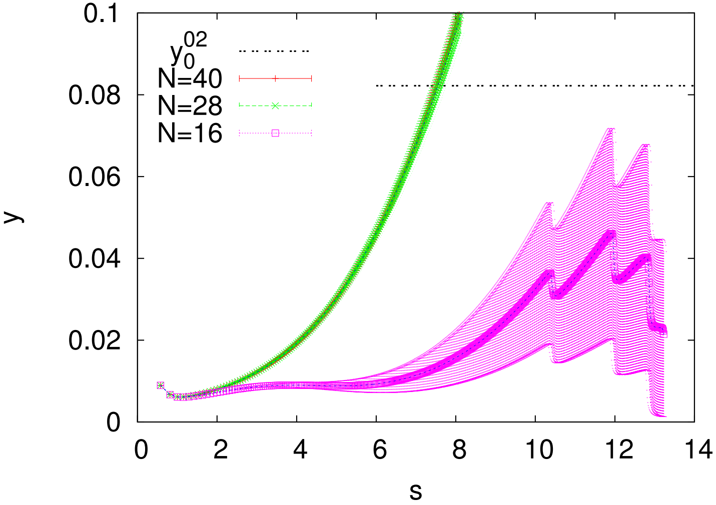
Backup: Energy operators

$$\langle U^{\square} \rangle = a_0 \mathbf{1} + i \sum_{j=1}^{3} a_j \sigma_j$$

- $E_0 \equiv 2\left[1 - a_0\right]$
- $E_1 \equiv \sum_{j=1}^{3} a_j^2$
- $E_4 \equiv \frac{1}{4} \sum_{j=1}^{3} \left(a_j^{(1)} + a_j^{(2)} + a_j^{(3)} + a_j^{(4)}\right)^2$

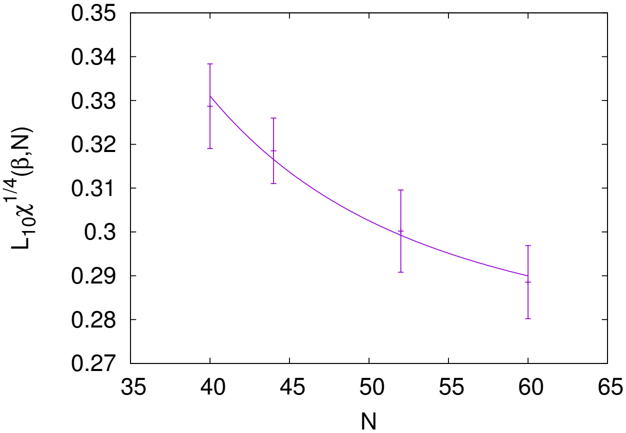


Backup: Disaster strikes!



Backup: Finite size extrapolation

$$L_{10}\chi^{1/4}(\beta, N) = L_{10}\chi^{1/4}(\beta) + \frac{c}{N^4}$$



Backup: Continuum limit extrapolation

$$L_{10}\chi^{1/4}(\beta) \approx L_{10}\chi^{1/4} + k\,a^2\Lambda_L^2 = L_{10}\chi^{1/4} + c\left(\frac{1}{L_{10}(\beta)}\right)^2$$