Energy-like observables in the chiral limit

D. A. Clarke, O. Kaczmarek, F. Karsch, A. Lahiri, M. Sarkar, C. Schmidt

Universität Bielefeld

Virtual CRC Meeting, 9 Dec 2020

A01

UNIVERSITÄT BIELEFELD

Fakultät für Physik

CRC–TR 211
Strong-interaction matter under extreme conditions

An entry point for discussion: the Polyakov loop

UNIVERSITÄT BIELEFELD
Fakultät für Physik

At $m = \infty$ with $N_c = 3$, the deconfinement order parameter is the Polyakov loop

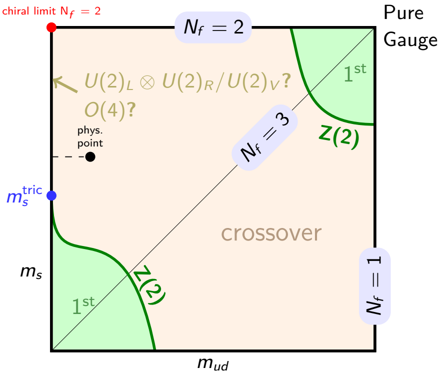
$$P_{\vec{x}} \equiv \frac{1}{3} \operatorname{tr} \prod_{\tau} U_4(\vec{x}, \tau), \qquad P \equiv \frac{1}{N_\sigma^3} \sum_{\vec{x}} P_{\vec{x}},$$

which relates to the color averaged quark-antiquark free energy

$$\exp\left[-\frac{F_{q\bar{q}}(r,T)}{T}\right] = \langle P_{\vec{x}} P_{\vec{y}}^{\dagger} \rangle \approx \langle P \rangle^2 \quad \text{(at large } r\text{)}.$$

Hence $\langle P \rangle = 0$ in the confined phase. In this phase $\langle P \rangle$ is invariant under global $\mathbb{Z}_3$, which otherwise transforms non-trivially as $P \to z\,P$. Spontaneous breaking above T_d.

The Polyakov loop in the Columbia plot



- Previous logic works in top-left corner.
- Can use $\langle P \rangle$ to study, e.g. order of transition[1] there.
- Situation is less clear in chiral limit...

[1] F. Cuteri et al., arXiv:2009.14033, (Sept. 2020).

The Polyakov loop in the chiral limit

- In the chiral limit, there is no clear global symmetry for deconfinement.
- Moreover P is purely gluonic, so P **is trivially invariant under chiral rotations**.
- Therefore, from the perspective of some $\mathcal{L}_{\text{eff}}$ written in the chiral limit, it should be an energy-like operator with respect to chiral transformations, and we may expect it to inherit behavior from the chiral transition in the same way any other energy-like operator would.

Let's explore this idea analytically and numerically.

Outline

1. Energy-like observables: The Polyakov loop
2. Energy-like observables: Conserved Charge fluctuations

Outline

1. Energy-like observables: The Polyakov loop[2,3]
2. Energy-like observables: Conserved Charge fluctuations

[2] D. A. Clarke et al., arXiv:2010.15825, (2020).
[3] D. A. Clarke et al., arXiv:2008.11678, (2020).

Some useful quantities

$H \equiv m_l/m_s$	symmetry breaking parameter
$t = (T - T_c)/T_c$	reduced temperature
$z \equiv z_0 t H^{-1/\beta\delta}$	scaling variable
$\langle P \rangle$	Polyakov loop
$F_q(T) = \lim_{r\to\infty} \frac{1}{2} F_{q\bar{q}}(r,T) = -T \log \langle P \rangle$	heavy quark free energy

$$\frac{1}{T}\frac{\partial F_q(T,H)}{\partial H} = -\frac{1}{\langle P \rangle}\frac{\partial \langle P \rangle}{\partial H}$$

F_q dependence on H

Being energy-like, P and F_q inherit singular behavior from $3d$ O(N) universality class[4]:

$$\frac{F_q}{T} = \underbrace{AH^{(1-\alpha)/\beta\delta} f_f'(z)}_{\text{singular part}} + \underbrace{f_{\rm reg}(T,H)}_{\text{regular part}}$$

with universal free energy scaling function f_f and critical exponents α, β, and δ. Prime indicates derivative w.r.t. z. In vicinity of chiral transition point, can expand $f_{\rm reg}$

$$f_{\rm reg}(T,H) = \sum_{ij} a^r_{i,2j}\; t^i\, H^{2j}.$$

We use O(2) since we will work with staggered quarks at fixed $N_\tau = 8$, so

$$\beta = 0.3490, \qquad \delta = 4.780, \qquad \alpha = -0.017 \qquad \Longrightarrow \qquad \frac{1-\alpha}{\beta\delta} = 0.61.$$

[4] J. Engels and F. Karsch, Phys. Rev. D, 85, 094506 (2012).

Derivatives with respect to H

Derivatives w.r.t. H will be more sensitive to H. Hence we compute

$$\frac{1}{T}\frac{\partial F_q(T,H)}{\partial H} = -AH^{(\beta-1)/\beta\delta}f_G'(z) + \frac{\partial}{\partial H}f_{\text{reg}}(T,H), \qquad \frac{\beta-1}{\beta\delta} = -0.39$$

where the order parameter scaling function f_G is related to f_f by

$$f_G(z) = -\left(1+\frac{1}{\delta}\right)f_f(z) + \frac{z}{\beta\delta}f_f'(z).$$

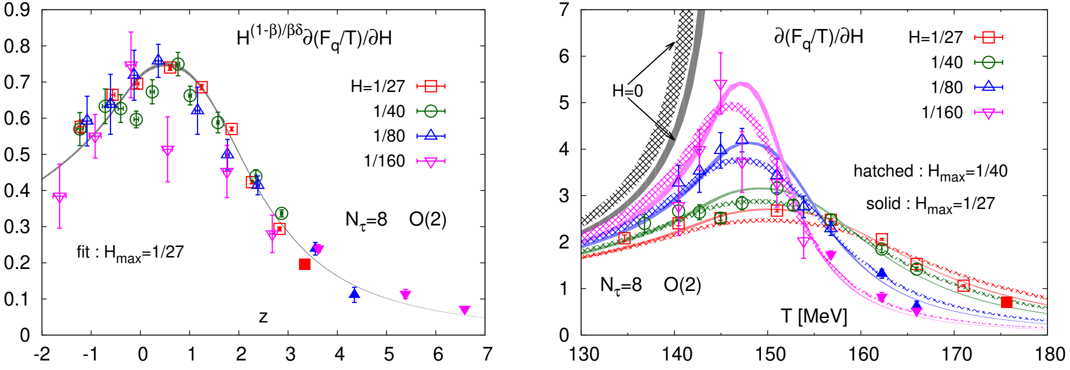
Expanding f_G' in z, setting $H=0$, one finds

$$\left.\frac{1}{T}\frac{\partial F_q(T,H)}{\partial H}\right|_{H=0} \sim \begin{cases} |t|^{\beta-1} & T<T_c \\ 0 & T>T_c \end{cases}.$$

Results: $\partial_H F_q$

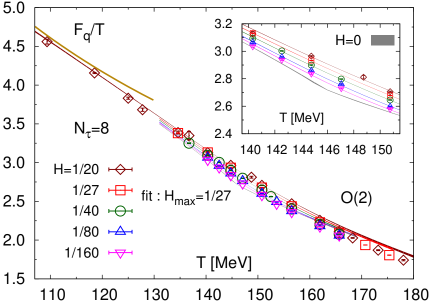
Singular part suggests 3-parameter fit:

$$H^{(1-\beta)/\beta\delta}\,\frac{1}{T}\frac{\partial F_q(T,H)}{\partial H} = -A\, f_G'\left(z_0\,\frac{T-T_c}{T_c}\,H^{-1/\beta\delta}\right)$$



Scaling plus regular behavior up to $\mathcal{O}(H^2)$ suggests 6-param fit (use A, z_0, T_c from before):

$$\frac{F_q(T,H)}{T} \approx AH^{(1-\alpha)/\beta\delta} f_f'\left(z_0 \frac{T-T_c}{T_c} H^{-1/\beta\delta}\right) + a^r_{0,0} + a^r_{1,0}t + a^r_{2,0}t^2$$

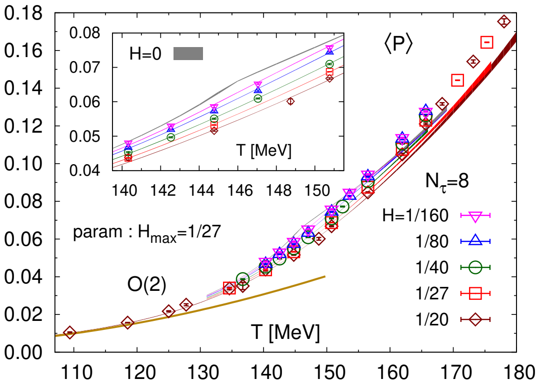


Solid gold line shows static-light meson contribution from HRG[5].

[5] A. Bazavov and P. Petreczky, Phys. Rev. D, 87.9, 094505 (2013).

$\langle P \rangle$ at many different quark masses

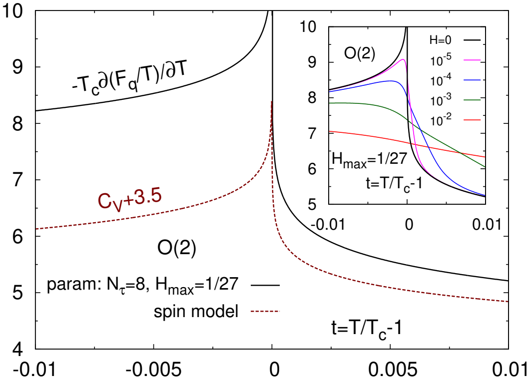
Using best fit parameters from above and evaluating $\langle P \rangle$:



Gold line: static-light meson contribution computed in HRG[6].

[6] A. Bazavov and P. Petreczky, Phys. Rev. D, 87.9, 094505 (2013).

Specific-heat-like observables



$$T_c \frac{\partial(F_q(T,0)/T)}{\partial T} = a^r_{1,0}\left(1 + R^{\pm}|t|^{-\alpha}\right)$$

Outline

1. Energy-like observables: The Polyakov loop
2. Energy-like observables: Conserved Charge fluctuations[7]

[7] M. Sarkar et al., arXiv:2011.00240, (Oct. 2020).

A small switch in notation

$$H \equiv m_l / m_s$$ symmetry breaking parameter

$$h \equiv H / h_0$$

$$t = \frac{1}{t_0}\left(\frac{T-T_c}{T_c} + \kappa_2^X\left(\frac{\mu_X}{T}\right)^2\right)$$ reduced temperature

$$z \equiv t h^{-1/\beta\delta}$$ scaling variable

$$\chi_{2n}^X = -\frac{\partial^{2n} f/T^4}{\partial(\mu_X/T)^{2n}}\bigg|_{\mu_X=0}$$ conserved charge fluctuations

Scaling, more generally

Again near the transition point we have[8]:

$$\frac{f(T,\vec{\mu},h)}{T^4} = \frac{1}{VT^3}\log Z(T,\vec{\mu},h) = \underbrace{h^{(2-\alpha)/\beta\delta} f_f(z)}_{\text{singular part}} + \underbrace{f_{\text{reg}}(T,\vec{\mu},h)}_{\text{regular part}},$$

The μ_X are energy-like couplings, and conserved charge fluctuations are derivatives of f w.r.t. the μ_X, and hence are **energy-like**. At $\vec{\mu}=0$ we find upon taking a derivative

$$\chi_2^X = -\frac{\partial^2 f/T^4}{\partial(\mu_X/T)^2}\bigg|_{\mu_X=0} = -2\kappa_2^X\, h^{(1-\alpha)/\beta\delta}\, f_f'(z) + \text{regular terms}.$$

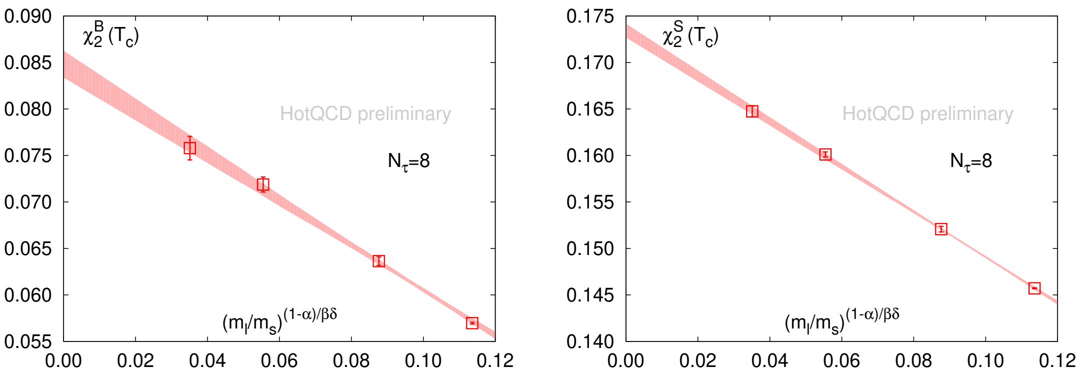
Again the O(2) exponents are

$$\beta = 0.3490, \qquad \delta = 4.780, \qquad \alpha = -0.017$$

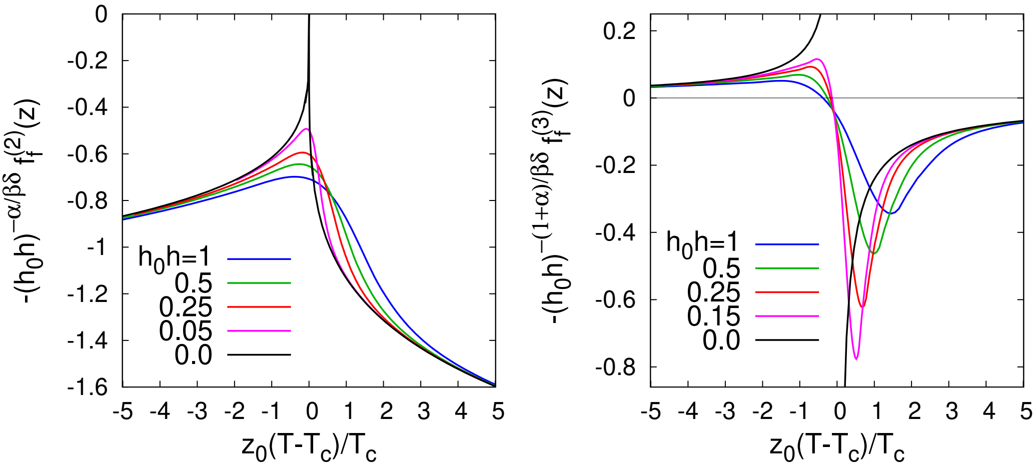
[8] J. Engels and F. Karsch, Phys. Rev. D, 85, 094506 (2012).

Results: Scaling of conserved charge fluctuations

$$\chi_2^X(t=0,h) \approx -B h^{(1-\alpha)/\beta\delta} + C_{reg}$$

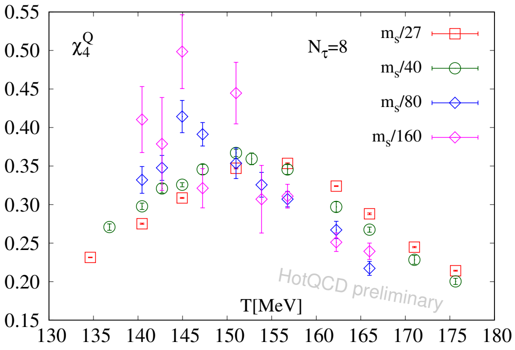


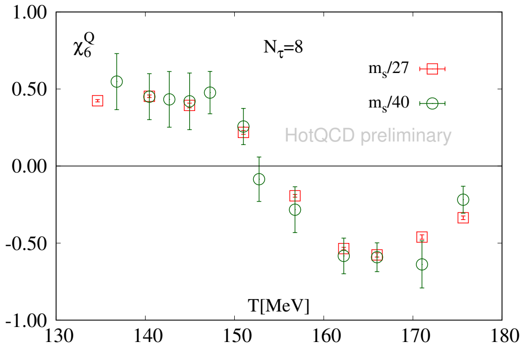
O(4) qualitative expectations[9]



[9] B. Friman et al., Eur. Phys. J. C, 71, 1694 (2011).

Results: Fourth and sixth order cumulants





Conclusions

Polyakov loop observables are sensitive to the chiral phase transition near the chiral limit.

At fixed N_τ:

- $\partial_H F_q$ diverges as $H \to 0$ according to the 3d O(2) universality class.
- $\langle P \rangle$ is described well by 3d O(2) scaling function near T_c.
- Specific-heat-like variables behave qualitatively similarly here as with O(N) spin model.
- Behavior of conserved charge fluctuations seemingly consistent with O(2) universality.

In the continuum limit:

- Highly plausible that it is consistent with O(4).

Thanks for your attention.