Sensitivity of the Polyakov Loop to Chiral Symmetry Restoration

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Global symmetry breaking

At $m=\infty$ with $N_c=3$, the deconfinement order parameter is the Polyakov loop

$$P_{\vec{x}} \equiv \frac{1}{3}\,\mathrm{tr}\prod_{\tau} U_4\left(\vec{x},\tau\right), \qquad P \equiv \frac{1}{N_\sigma^3}\sum_{\vec{x}} P_{\vec{x}},$$

which relates to the color averaged quark-antiquark free energy

$$\exp\left[-\frac{F_{q\bar{q}}(r,T)}{T}\right] = \langle P_{\vec{x}} P_{\vec{y}}^{\dagger}\rangle \approx \langle \mathrm{Re}\,P\rangle^2 \quad \text{(at large } r\text{)}.$$

Hence $\langle \mathrm{Re}\,P\rangle = 0$ in the confined phase. In this phase $\langle \mathrm{Re}\,P\rangle$ is invariant under global $\mathbb{Z}_3$, which otherwise transforms non-trivially as $P \to z\,P$. Spontaneous breaking above T_d.

At $m=0$ the chiral condensate $\langle\bar{\psi}\psi\rangle$ transforms non-trivially under $SU(2)_A$. Hence $\langle\bar{\psi}\psi\rangle > 0$ signals chiral symmetry breaking. Spontaneous breaking below T_c.

The original question

What are good observables that indicate deconfinement in QCD?

In pure SU(3) gauge theory, inflection points found at similar locations[1].

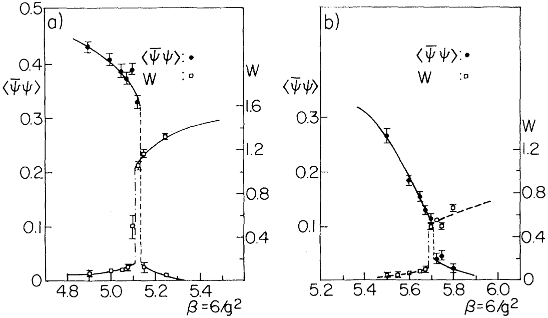


FIG. 2. $\langle\bar\psi\psi\rangle$ and W vs $\beta=6/g^2$ for SU(3) gauge theory on (a) 2×8^3 and (b) 4×8^3 lattices.

[1] J. Kogut et al., Phys. Rev. Lett. 50.6, 393–396 (1983).

At larger-than-physical quark mass

Similar locations[2] also for $N_f = 2 + 1$, physical m_s, and $m_\pi \approx 220$ [MeV].

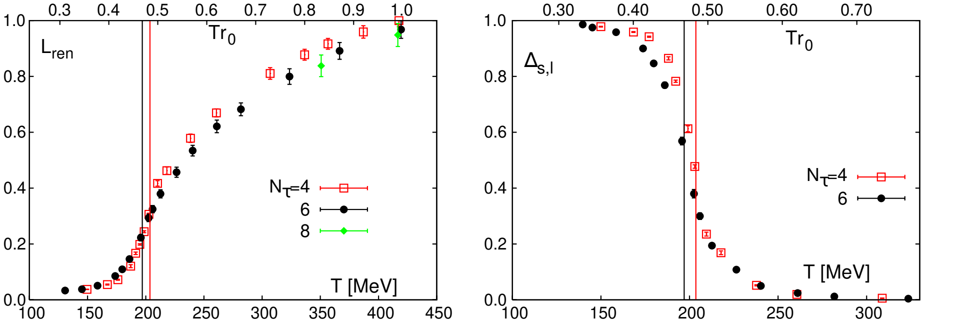
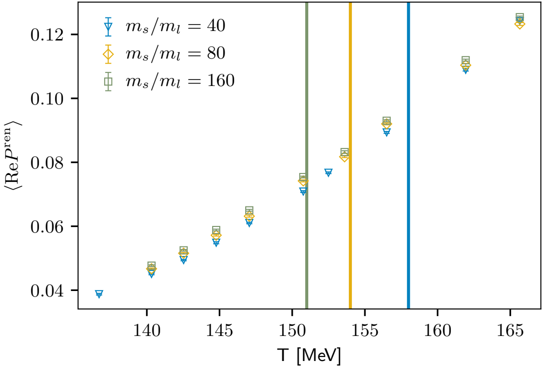


FIG. 11 (color online). Renormalized Polyakov loop on lattices with temporal extent N 4, 6 and 8 (left) and the normalized difference of light and strange quark chiral condensates def ned in Eq. (36). The vertical lines show the location of the transition temperature determined in [18] on lattices with temporal extent N 4 (right line) and in this analysis for N 6 (left line).

[2] M. Cheng et al., Phys. Rev. D, 77.1, 014511 (2008).

At smaller-than-physical quark mass

No longer the case for highly improved fermion actions, and at lower quark masses[3]. Vertical lines indicate chiral T_{pc} locations[4].



[3]D. A. Clarke et al., arXiv:1911.07668 [hep-lat], (2019).
[4]H. Ding et al., Phys. Rev. Lett. 123.6, 062002 (2019).

How can we understand this behavior?

- At $m < \infty$, $\langle P \rangle$ still has inflection point somewhere.
- This is (often?) interpreted as some remnant of the $m = \infty$ critical behavior.
- But in the chiral limit, there is no clear global symmetry for deconfinement.
- Moreover P is purely gluonic, so P **is trivially invariant under chiral rotations**.
- Therefore, from the perspective of some $\mathcal{L}_{\text{eff}}$ written in the chiral limit, it should be an energy-like operator with respect to chiral transformations, and we may expect it to inherit behavior from the chiral transition in the same way any other energy-like operator would.

Let's explore this idea analytically and numerically.

Some useful quantities

$H \equiv m_l/m_s$	symmetry breaking parameter
$t = (T - T_c)/T_c$	reduced temperature
$z \equiv z_0 t H^{-1/\beta\delta}$	scaling variable
$\langle \mathrm{Re}\, P \rangle$	Polyakov loop
$F_q(T) = \lim_{r\to\infty} \frac{1}{2} F_{q\bar{q}}(r,T) = -T \log \langle \mathrm{Re}\, P \rangle$	heavy quark free energy

$$\frac{1}{T}\frac{\partial F_q(T,H)}{\partial H} = -\frac{1}{\langle \mathrm{Re}\, P \rangle}\frac{\partial \langle \mathrm{Re}\, P \rangle}{\partial H}$$

F_q dependence on H

Being energy-like, P and F_q inherit singular behavior from 3d O(N) universality class[5]:

$$\frac{F_q}{T} = \underbrace{AH^{(1-\alpha)/\beta\delta} f_f'(z)}_{\text{singular part}} + \underbrace{f_{\text{reg}}(T, H)}_{\text{regular part}}$$

with universal free energy scaling function f_f and critical exponents α, β, and δ. Prime indicates derivative w.r.t. z. In vicinity of chiral transition point, can expand f_{reg}

$$f_{\text{reg}}(T, H) = \sum_{ij} a^r_{i,2j}\, t^i H^{2j}.$$

We use O(2) since we will work at fixed $N_\tau = 8$, so

$$\beta = 0.3490, \qquad \delta = 4.780, \qquad \alpha = -0.017.$$

[5] J. Engels and F. Karsch, Phys. Rev. D, 85, 094506 (2012).

F_q dependence on H

$$\frac{F_q}{T} = AH^{(1-\alpha)/\beta\delta} f_f'(z) + f_{\mathrm{reg}}(T,H)$$

In $H \to 0$ limit, keeping only leading terms[6]:

$$\frac{F_q}{T} \sim \begin{cases} a^-(T) + Ap_s^-(T)\ H & T < T_c \\ a^r_{0,0} + Ap_0\ H^{(1-\alpha)/\beta\delta} & T = T_c \\ a^+(T) + p^+(T)\ H^2 & T > T_c \end{cases}$$

$$\frac{1-\alpha}{\beta\delta} = 0.61$$

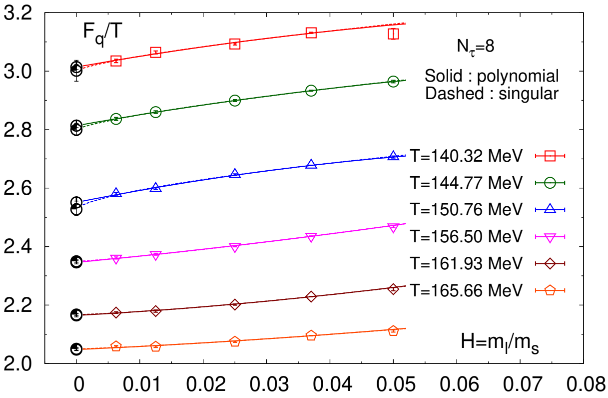
Above equations will suggest our fitting forms.

[6] J. Engels and F. Karsch, Phys. Rev. D, 85, 094506 (2012).

- $N_f = 2 + 1$ with HISQ action
- $N_\tau = 8$
- $N_s / N_\tau \geq 4$
- m_s fixed to its physical value
- m_s / m_ℓ varies from 20 to 160 (160 MeV $\gtrsim m_\pi \gtrsim$ 58 MeV)
- T in the vicinity of chiral crossover
- New configurations, also configurations from past studies[7,8]
- Renormalization constants, when needed, from TUMQCD[9]

[7] A. Bazavov et al., Phys. Lett. B, 795, 15–21 (2019).
[8] H. Ding et al., Phys. Rev. Lett. 123.6, 062002 (2019).
[9] A. Bazavov et al., Phys. Rev. D, 93.11, 114502 (2016).

Results: F_q dependence on H (at fixed T)



General form

$$\frac{F_q}{T} \sim \begin{cases} a^-(T) + Ap_s^-(T)\ H & T < T_c \\ a^r_{0,0} + Ap_0\ H^{(1-\alpha)/\beta\delta} & T = T_c \\ a^+(T) + p^+(T)\ H^2 & T > T_c \end{cases}$$

suggests 3-parameter fits

$$F_q^{\mathrm{sin}}(H) = a + bH^c,$$
$$F_q^{\mathrm{poly}}(H) = a + bH + cH^2.$$

Former fit near $T_c^{N_\tau=8} \approx 144$ [MeV]:

$$c = \begin{cases} 0.7(2) & T = 144.77\ [\mathrm{MeV}] \\ 0.6(1) & T = 150.76\ [\mathrm{MeV}]. \end{cases}$$

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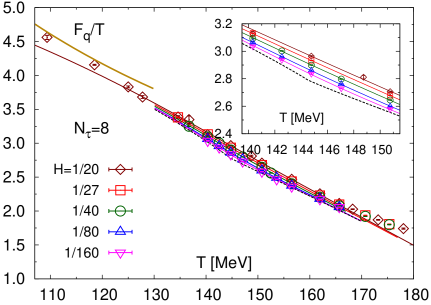
Scaling plus regular behavior up to $\mathcal{O}(H^2)$ suggests a 6-parameter fit:

$$\frac{F_q(T,H)}{T} = A H^{(1-\alpha)/\beta\delta} f_f'\left(z_0 \frac{T-T_c}{T_c} H^{-1/\beta\delta}\right) + a^r_{0,0} + a^r_{1,0}t + a^r_{2,0}t^2$$



Solid gold line shows static-light meson contribution from HRG[10].

[10] A. Bazavov and P. Petreczky, Phys. Rev. D, 87.9, 094505 (2013).

Derivatives with respect to H

Derivatives of observables w.r.t. H will be more sensitive to H. Hence we compute

$$\frac{1}{T}\frac{\partial F_q(T,H)}{\partial H} = -AH^{(\beta-1)/\beta\delta} f_G'(z) + \frac{\partial}{\partial H} f_{\text{reg}}(T,H), \qquad \frac{\beta-1}{\beta\delta} = -0.39$$

where the order parameter scaling function f_G is related to f_f by

$$f_G(z) = -\left(1+\frac{1}{\delta}\right) f_f(z) + \frac{z}{\beta\delta} f_f'(z).$$

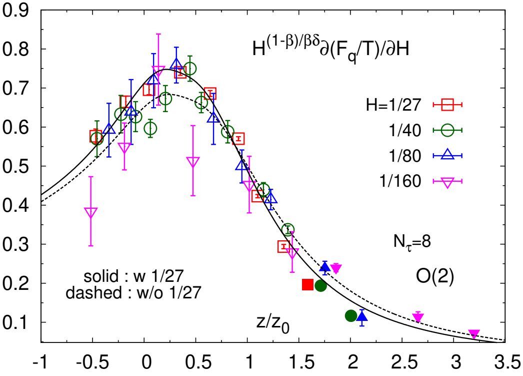
Expanding f_G' in z, setting $H=0$, one finds

$$\left.\frac{1}{T}\frac{\partial F_q(T,H)}{\partial H}\right|_{H=0} \sim \begin{cases} |t|^{\beta-1} & T < T_c \\ 0 & T > T_c \end{cases}.$$

Results: Rescaled $\partial_H F_q$

Singular part suggests 3-parameter fits

$$H^{(1-\beta)/\beta\delta}\,\frac{1}{T}\,\frac{\partial F_q(T,H)}{\partial H} = -A\, f'_G\left(z_0\,\frac{T-T_c}{T_c}\,H^{-1/\beta\delta}\right)$$

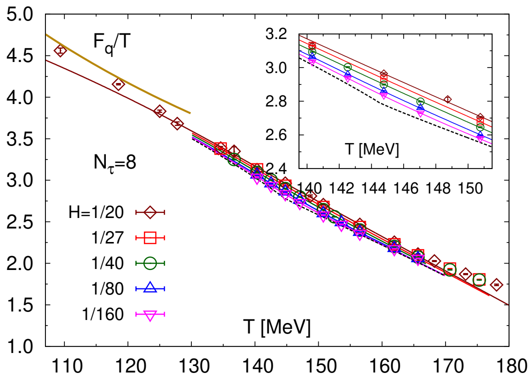


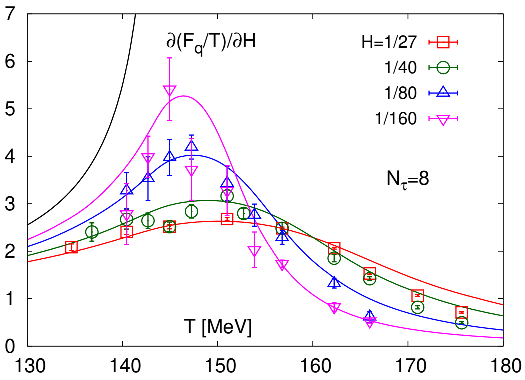
	A	T_c	z_0	χ^2/d.o.f.
dash	2.27(5)	144.3(6)	1.85(9)	1.40
solid	2.48(4)	145.5(5)	2.24(8)	4.86

Compare[11] with $T_c^{N_\tau=8} = 144(2)$ [MeV].

[11] H. Ding et al., Phys. Rev. Lett. 123.6, 062002 (2019).

Results: $\partial_H F_q$



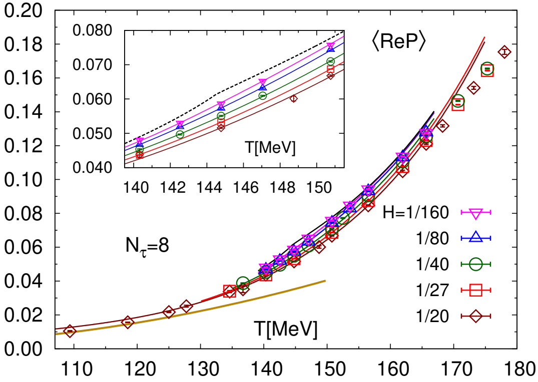


Singular part fit parameters shared by fit for F_q. Joint fit yields:

A	T_c	z_0	χ^2/d.o.f.
2.41(3)	144.4(6)	1.82(27)	1.92

⟨Re P⟩ at many different quark masses

Using best fit parameters from above and evaluating ⟨Re P⟩:



Gold line: static-light meson contribution computed in HRG[12].

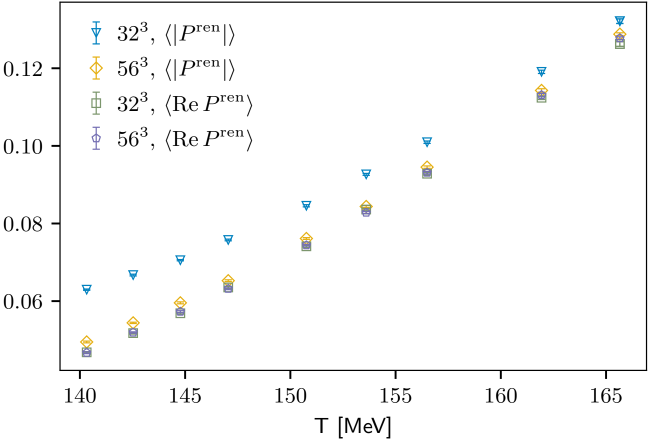
[12] A. Bazavov and P. Petreczky, Phys. Rev. D, 87.9, 094505 (2013).

Summary

- Polyakov loop observables are sensitive to the chiral phase transition near the chiral limit.
- In particular $\partial_H F_q$ diverges as $H \to 0$ according to the $3d$ O(2) universality class.
- $\langle P \rangle$ is described well by $3d$ O(2) scaling function near T_c.

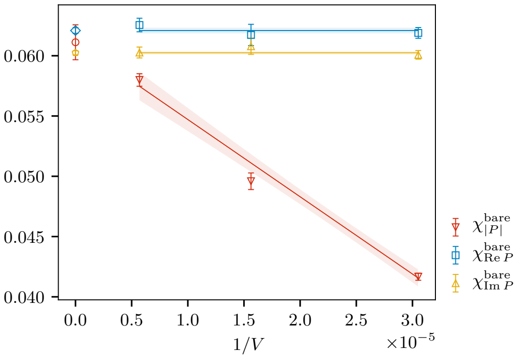
Thanks for your attention.

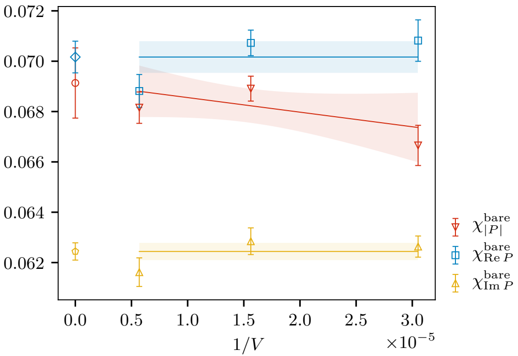
N_σ dependence of P. $N_\tau = 8$ and $m_s/m_l = 80$ for these.



Additional details: χ_P finite size scaling

Finite size scaling of various susceptibilities for $T \approx 140$ [MeV] (left) and $T \approx 165$ [MeV] (right). $N_\tau = 8$ and $m_s/m_l = 80$ for these.





Additional details: current statistics

m_s/m_l	N_σ	avg. # TU
20	32	99 000
27	32	1 500 000
40	40	110 000
80	56	35 000
	40	33 000
	32	73 000
160	56	17 000