Imprint of Chiral Symmetry Restoration on the Polyakov Loop and Heavy Quark Free Energy

D. Clarke, O. Kaczmarek, F. Karsch, A. Lahiri, M. Sarkar

Universität Bielefeld

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UNIVERSITÄT BIELEFELD
Fakultät für Physik

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Strong-interaction matter under extreme conditions

The Polyakov loop in the quenched limit

$m = \infty$: deconfinement order parameter is Polyakov loop,

$$P_{\vec{x}} \equiv \frac{1}{N_c} \operatorname{tr} \prod_{\tau} U_4(\vec{x}, \tau), \qquad P \equiv \frac{1}{N_\sigma^3} \sum_{\vec{x}} P_{\vec{x}}.$$

Related to static quark-antiquark free energy,

$$\exp\left[-\frac{F_{q\bar{q}}(r,T)}{T}\right] = \langle P_{\vec{x}} P_{\vec{y}}^{\dagger} \rangle \approx \langle P \rangle^2 \quad \text{(large } r\text{)}.$$

Standard story:

- $\langle P \rangle = 0$ confined.
- $\langle P \rangle \neq 0$ deconfined.
- Spontaneous $\mathbb{Z}_3$ breaking above T_d.

Seeming coincidence[1] of inflection points not seen closer to continuum limit[2,3,4] and with physical or smaller m_l.

How to interpret Polyakov loop?

What influences its behavior, especially near chiral transition?

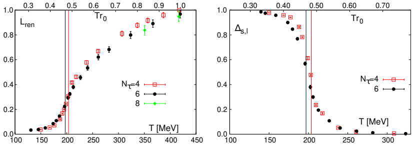
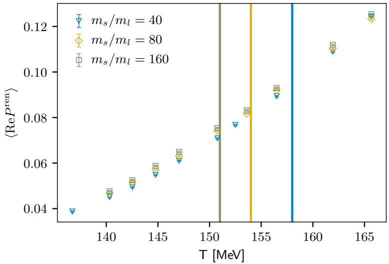


FIG. 11 (color online). Renormalized Polyakov loop on lattices with temporal extent N 4, 6 and 8 (left) and the normalized difference of light and strange quark chiral condensates def ned in Eq. (36). The vertical lines show the location of the transition temperature determined in [18] on lattices with temporal extent N 4 (right line) and in this analysis for N 6 (left line).



[1] M. Cheng et al., Phys. Rev. D, 77.1, 014511 (2008).
[2] A. Bazavov et al., Phys. Rev. D, 93.11, 114502 (2016).
[3] D. Clarke et al., PoS(LATTICE2019), 194 (2020).
[4] H. Ding et al., Phys. Rev. Lett. 123.6, 062002 (2019).

The Polyakov loop in the chiral limit

- Chiral limit: **no clear global symmetry** for deconfinement.
- P is purely gluonic $\Rightarrow$ P **invariant under chiral rotations**.
- Take RG perspective near chiral transition point:
  - P is energy-like[5] (effective coupling respects chiral symmetry).
  - Inherits behavior from energy-like scaling functions.
  - Chiral scaling in context: Anirban's talk.
  - More energy-like operators: Mugdha's talk.

Explore this idea analytically and numerically.

Simulation parameters:

- $N_f = 2 + 1$ HISQ action
- m_s fixed at physical point
- $N_s/N_\tau \geq 3$

[5] We are borrowing nomenclature from spin systems. Operators coupling to the symmetry-breaking parameter are magnetization-like.

Outline

1. Energy-like observables: $\langle P \rangle$ and F_q
2. C_V-like observable: Response of F_q to μ_B

1. Energy-like observables: $\langle P \rangle$ and F_q[6,7]
2. C_V-like observable: Response of F_q to μ_B

[6] D. Clarke et al., Phys. Rev. D, 103.1, L011501 (2021).
[7] D. Clarke et al., Acta Phys. Pol. B Proc. Suppl. 14.2, 311 (2021).

RG parameters and observables of interest

$H \equiv m_l/m_s$	symmetry breaking param.
$t = (T - T_c)/T_c$	reduced temperature
$z \equiv z_0 t H^{-1/\beta\delta}$	scaling variable

$\langle P \rangle$	Polyakov loop
$F_q \equiv -T \log \langle P \rangle$	heavy quark free energy
$\frac{1}{T}\frac{\partial F_q}{\partial H} = -\frac{1}{\langle P \rangle}\frac{\partial \langle P \rangle}{\partial H}$	its derivatives[8,9]
$T_c \frac{\partial F_q/T}{\partial T} = -\frac{T_c}{\langle P \rangle}\frac{\partial \langle P \rangle}{\partial T}$	

[8] Expected to be independent of renormalization.

[9] We sometimes refer to $\partial_H F_q$ as a mixed susceptibility.

F_q dependence on H

Assumption: P and F_q inherit behavior from $3d$ $\mathrm{O}(N)$ universality class[10]:

$$\frac{F_q}{T} = \underbrace{AH^{(1-\alpha)/\beta\delta} f_f'(z)}_{\text{singular part}} + \underbrace{f_{\text{reg}}(T,H)}_{\text{regular part}}$$

with universal free energy scaling function f_f and critical exponents α, β, and δ. Prime is derivative w.r.t. z. Can expand f_{reg}

$$f_{\text{reg}}(T,H) = \sum_{i\,j} a^r_{i,2j}\; t^i\, H^{2j}.$$

HISQ quarks at fixed N_τ $\Rightarrow$ use $\mathrm{O}(2)$ exponents

$$\beta = 0.3490, \qquad \delta = 4.780, \qquad \alpha = -0.017 \qquad \Rightarrow \qquad \frac{1-\alpha}{\beta\delta} = 0.61.$$

[10] J. Engels and F. Karsch, Phys. Rev. D, 85, 094506 (2012).

Derivatives w.r.t. H more sensitive to H:

$$\frac{1}{T}\partial_H F_q(T,H) = -AH^{(\beta-1)/\beta\delta} f_G'(z) + \partial_H f_{\rm reg}(T,H), \qquad \frac{\beta-1}{\beta\delta} = -0.39$$

with order parameter scaling function f_G

$$f_G(z) = -\left(1+\frac{1}{\delta}\right) f_f(z) + \frac{z}{\beta\delta} f_f'(z).$$

Expanding f_G' in z, setting $H=0$, one finds

$$\frac{1}{T}\partial_H F_q(T,H)\bigg|_{H=0} \sim \begin{cases} |t|^{\beta-1} & T<T_c \\ 0 & T>T_c \end{cases}.$$

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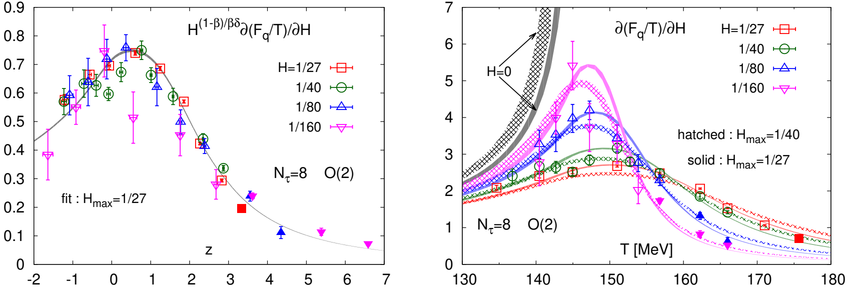
Singular part suggests 3-parameter fit:

$$H^{(1-\beta)/\beta\delta}\,\frac{1}{T}\,\frac{\partial F_q(T,H)}{\partial H} = -A\, f_G'\left(z_0\,\frac{T-T_c}{T_c}\,H^{-1/\beta\delta}\right)$$

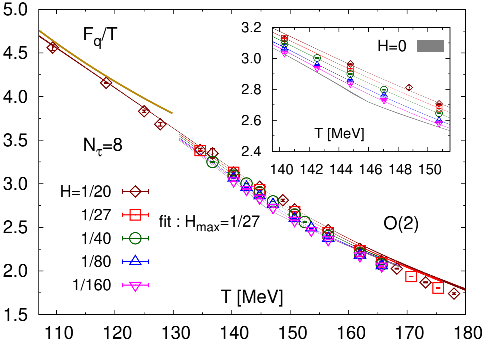


H-derivative diverges as expected from scaling.
T_c and z_0 consistent with previous determinations[11].

[11] H. Ding et al., Phys. Rev. Lett. 123.6, 062002 (2019).

Scaling plus regular behavior up to $\mathcal{O}(H^2)$ suggests 6-param fit:

$$\frac{F_q(T,H)}{T} \approx AH^{(1-\alpha)/\beta\delta} f_f'\left(z_0 \frac{T-T_c}{T_c} H^{-1/\beta\delta}\right) + a^r_{0,0} + a^r_{1,0}t + a^r_{2,0}t^2$$

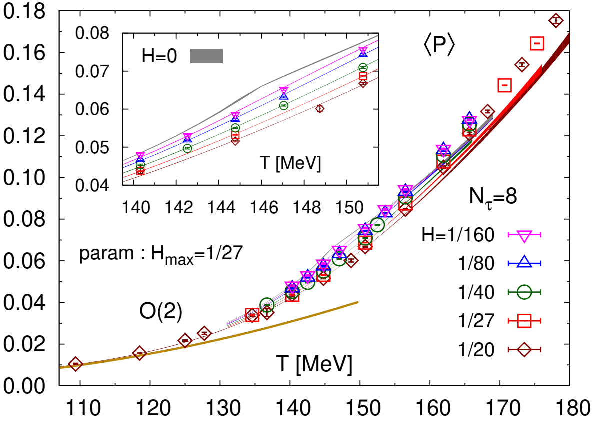


Gold line: static-light meson contribution computed in HRG[12].

[12] A. Bazavov and P. Petreczky, Phys. Rev. D, 87.9, 094505 (2013).

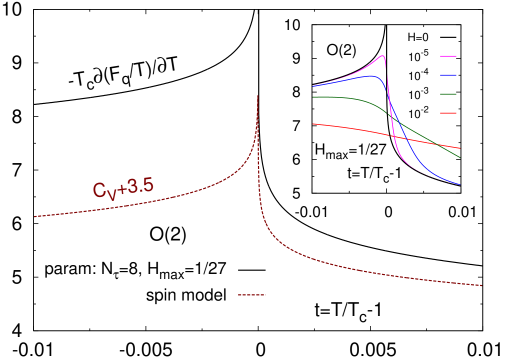
Results: $\langle P \rangle$ at many different quark masses

Use best fit parameters from above and plug into $\langle P \rangle = e^{-F_q/T}$:



$\langle P \rangle$ behavior in this (T, m_l) regime follows chiral scaling!

Specific-heat-like observables



$$T_c \frac{\partial (F_q(T,0)/T)}{\partial T} = a^r_{1,0}\left(1 + R^{\pm}|t|^{-\alpha}\right)$$

Spike in $\partial_T F_q$ in chiral limit: C_V-like.

1. Energy-like observables: $\langle P \rangle$ and F_q
2. C_V-like observable: Response of F_q to μ_B

Including chemical potential in our RG analysis

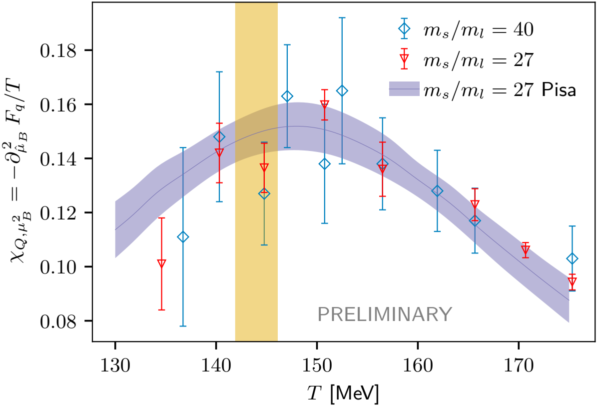
μ is energy-like coupling. Can include in general energy-like coupling:

$$t \equiv \frac{1}{t_0}\left(\frac{T-T_c}{T_c}+\kappa\left(\frac{\mu}{T}\right)^2\right)$$

With $\hat{\mu}\equiv\mu/T$ this leads to expectation

$$\chi_{Q,\mu_B^2}\equiv-\partial^2_{\hat{\mu}_B}\,F_q/T\,\Big|_{\hat{\mu}_B=0}$$

should also be C_V-like.



Physical m_l agrees well with literature[13,14].

$m_s/m_l = 40$ requires higher statistics.

[13] M. D'Elia et al., Phys. Rev. D, 100.5, 054504 (2019).

[14] Vertical yellow band indicates T_c for $N_\tau = 8$.

Conclusions

$\langle P \rangle$ observables are sensitive to chiral phase transition near critical point!

At fixed N_τ:

- $\partial_H F_q$ diverges as $H \to 0$ according to the $3d$ O(2) universality class.
- $\langle P \rangle$ is described well by $3d$ O(2) scaling function near T_c.
- $\partial_T F_q$ behaves qualitatively similarly as C_V in O(2) spin model.
- In process of investigating chiral behavior of χ_{Q,μ_B^2}.

In the continuum limit:

- Highly plausible that it is consistent with O(4).

Thanks for your attention.