QCD material parameters at non-zero chemical potential from the lattice

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Motivation and strategy

Useful and interesting to have EoS at $\mu_B > 0$. But we only have direct access to $\mu_B = 0$ on the lattice. Commonly played game:

1. Write p/T^4 as **Taylor expansion**[1] in μ_i/T
2. Derive all other observables from P/T^4 using **thermodynamics**
3. **Measure** Taylor coefficients on lattice
4. Compare against **HRG** at low T
5. Compare against **ideal gas** or **perturbation theory** at large T

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[1] See e.g. Borsányi Tue 14:50; Pásztor plenary.

Lattice approach

For convenience, $\hat{X} \equiv XT^{-k}$ with k s.t. $\hat{X}$ dimensionless (e.g. $\hat{\mu} = \mu/T$)

Dealing with 3 chemical potentials $\hat{\mu}_B$, $\hat{\mu}_Q$, $\hat{\mu}_S$

To make contact with T-$\hat{\mu}_B$ plane, need to eliminate 2 independent variables

Can **impose external constraint**, e.g.

1. $\hat{\mu}_Q = \hat{\mu}_S = 0$
2. $n_S = 0$, $n_Q/n_B = 0.4$ (RHIC-like)
3. $n_S = 0$, $n_Q/n_B = 0.5$ (isospin-symmetric; yields $\hat{\mu}_Q = 0$)

and think of **expansions in $\hat{\mu}_B$ only**:

$$\hat{p} = \sum_{i,j,k=0}^{\infty} \frac{\chi_{ijk}^{BQS}}{i!j!k!} \hat{\mu}_B^i \hat{\mu}_Q^j \hat{\mu}_S^k \qquad \rightarrow \qquad \hat{p} = \sum_{k \text{ even}} \frac{P_k(T)}{k!} \hat{\mu}_B^k$$

Hadron resonance gas (HRG): the basics

Non-interacting, quantum, relativistic gas eventually gives (single species)

$$p = \frac{m^2 g T^2}{2\pi^2} \sum_{k=1}^{\infty} \frac{\eta^{k+1} z^k}{k^2} K_2\left(\frac{mk}{T}\right), \qquad z \equiv e^{\hat{\mu}_B B + \hat{\mu}_Q Q + \hat{\mu}_S S},$$

with K_2 modified Bessel function $2^{\rm nd}$ kind. HRG:

- Assume such gas where hadrons and resonances only d.o.f.
- Hence valid up to $\sim T_{\rm pc}$
- Sum over all such states, each with g_i, m_i, etc.
- K_2 exponentially suppressed, so can keep few terms

Some context and lattice setup

Related studies from the past, for instance $\hat{\mu}_B = 0$[2,3] and $\hat{\mu}_B > 0$[4,5].

This study:

- **High statistics** with $\lesssim 1.5\text{M}$ configurations per ensemble.
- **Continuum extrapolated** for $\mathcal{O}\left(\hat{\mu}_B^2\right)$ and $\mathcal{O}\left(\hat{\mu}_B^4\right)$ coefficients.
- Taylor series up to 8^{th} **order**; converges well at least for $\hat{\mu}_B < 1.5$[6,7].
- Use these coefficients to construct EoS[8].

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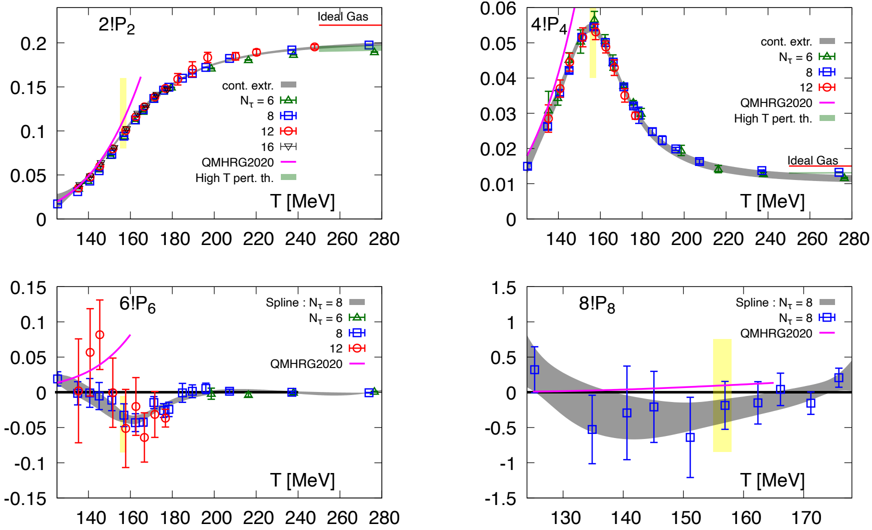
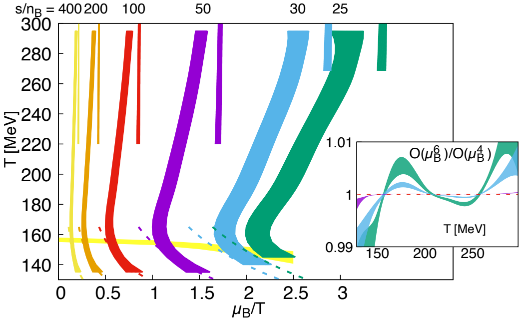
[2] A. Bazavov et al., Phys. Rev. D, 90, 094503 (2014).
[3] S. Borsányi et al., Physics Letters B, 730, 99–104 (2014).
[4] A. Bazavov et al., Phys. Rev. D, 95.5, 054504 (2017).
[5] S. Borsanyi et al., J. High Energ. Phys. 2018.10, 205 (2018).
[6] D. Bollweg et al., Phys. Rev. D, 105.7, 074511 (2022).
[7] Strictly speaking, convergence radius depends on temperature.
[8] D. Bollweg et al., Phys. Rev. D, 108.1, 014510 (2023).

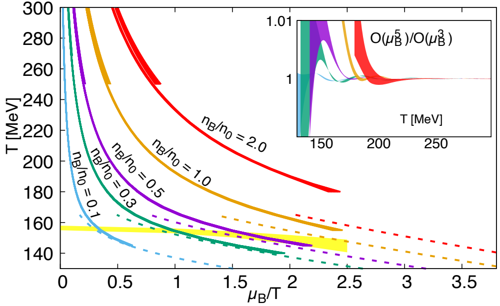
Results: HotQCD pressure coefficients[9]



[9] D. Bollweg et al., Phys. Rev. D, 108.1, 014510 (2023).

Results: Lines of constant physics[10] $n_S = 0,\ n_Q/n_S = 0.5$





Results from $\mathcal{O}(\hat{\mu}_B^6)$ pressure series, high reliability.

Good agreement between lattice and HRG below $T_{\rm pc}$.

Approaches $\mathcal{O}(g^2)$ perturbation theory at high T.

[10] D. Bollweg et al., Phys. Rev. D, 108.1, 014510 (2023).

Material parameter: Speed of sound

$$c_T^2 = \left(\frac{\partial p}{\partial \epsilon}\right)_T \qquad c_s^2 = \left(\frac{\partial p}{\partial \epsilon}\right)_{s/n_B} = \left(\frac{\partial p/\partial T}{\partial \epsilon/\partial T}\right)_{s/n_B}$$

"Trick" on RHS to get c_s^2; **now take more direct approach**.

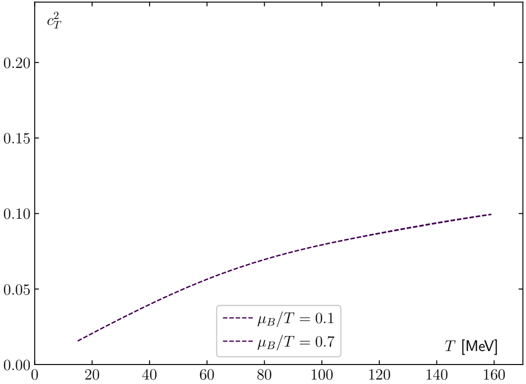
Can use c_s^2 to learn about QCD phase diagram:

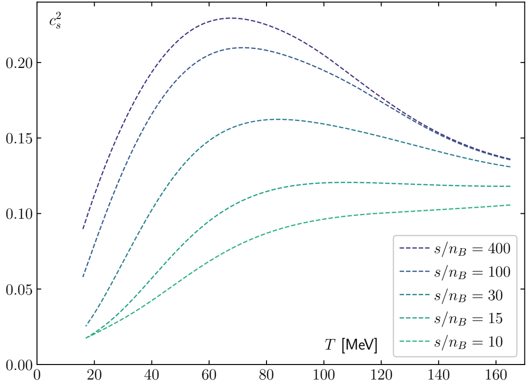
- Related to cooling/expansion rate
- Can be related to bulk viscosity at high T[11]
- Minimum/"softest point" may indicate long-lived fireball[12]

c_T^2 in principle[13] accessible in HIC.

[11] P. B. Arnold, C. Dogan, and G. D. Moore, Phys. Rev. D, 74, 085021 (2006).
[12] C. M. Hung and E. V. Shuryak, Phys. Rev. Lett. 75, 4003–4006 (1995).
[13] A. Sorensen et al., Phys. Rev. Lett. 127.4, 042303 (2021).

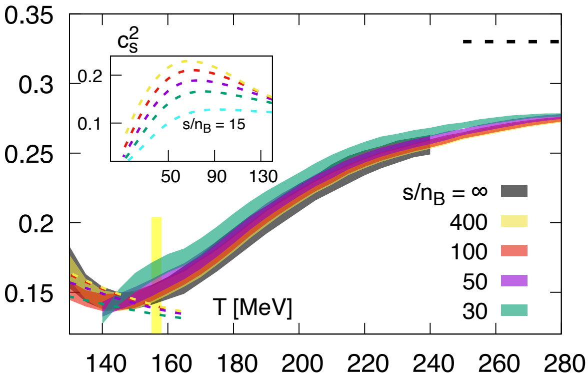
Results: HRG c_X^2 for $n_S = 0$, $\hat{\mu}_Q = 0$





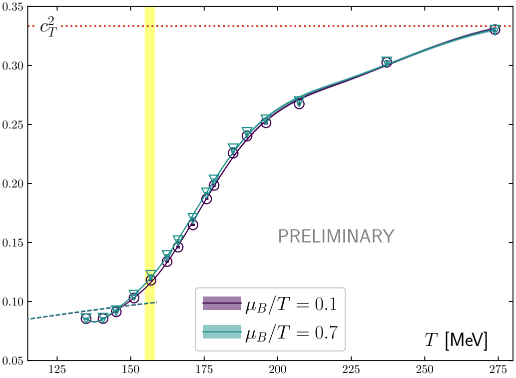
$$\hat{\mu}_Q = \hat{\mu}_S = 0 \quad \text{HRG:} \quad c_T^2 = \left(\frac{\partial p}{\partial \hat{\mu}_B}\right)\left(\frac{\partial \epsilon}{\partial \hat{\mu}_B}\right)^{-1} = \frac{1}{3 + f_B'(T)/f_B(T)}$$

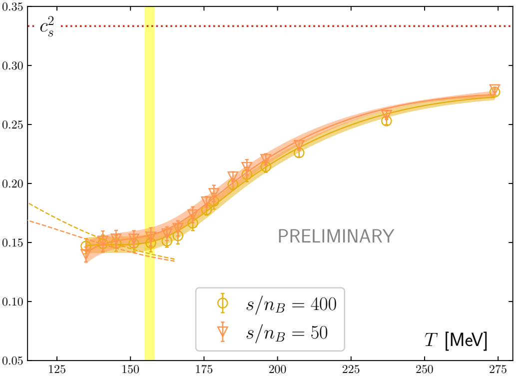
Bump in c_s^2 related to interplay of mesons and baryons; c_T^2 determined by baryons so no bump.



[14] D. Bollweg et al., Phys. Rev. D, 108.1, 014510 (2023).

Results: "Direct" Lattice c_X^2 for $n_S = 0$, $\hat{\mu}_Q = 0$

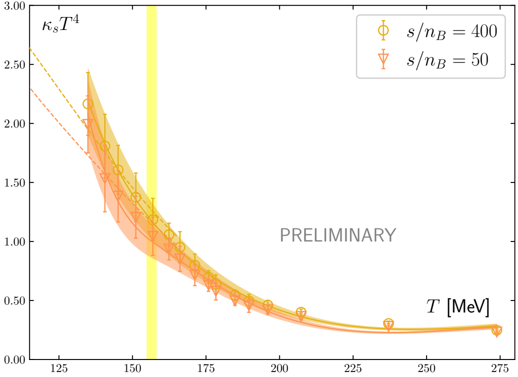




Dip in c_s^2 in vicinity of $T_{\rm pc}$.
Dependence of c_s^2 on s/n_B is at most mild.

Material parameter: Adiabatic compressibility for $n_S = 0$, $\hat{\mu}_Q = 0$

$$\kappa_s = \frac{1}{n_B}\left(\frac{\partial n_B}{\partial p}\right)_{s/n_B}$$



Formally fulfills $\kappa_s = 1/c_s^2(\epsilon + p - \mu_Q n_Q - \mu_S n_S)$.

Conclusions

- Have QCD EoS at finite $\hat{\mu}_B$ for $n_S = \hat{\mu}_Q = 0$ systems; lowest orders continuum-extrapolated.
- Dip in c_s^2 can be attributed to mesonic contribution in HRG.
- Dependence of these observables on $\hat{\mu}_B$ seems to be mild.
- Should be able to get α, C_V, and C_p.

Thanks for your attention.